

Force-Frequency Coefficient of Singly Rotated Vibrating Quartz Crystals

Abstract: Frequency perturbations in vibrating quartz crystals, caused by externally applied forces, have been investigated for some time. The parameters affecting the force-frequency sensitivity were recently established and evaluated, making possible the derivation of a normalized coefficient K_f . An empirical chart, showing the coefficient K_f for all singly rotated crystals, $(y \times l) \theta$, is presented for all directions of force in the azimuth angle ψ . The maximum force-frequency coefficient is found to be $|K_f| \cong 30 \times 10^{-15} (msN^{-1})$ for angles $\theta = +17^\circ, +26.5^\circ$ and $+64^\circ$. A crystal cut $(y \times l) -18.5^\circ$ shows a constant coefficient $K_f = +5.3 \times 10^{-15} (msN^{-1})$ for all angles of ψ . Coefficients $K_f(\psi)$ for BT-, Y- and AT- cut crystals are also shown on separate graphs.

The force-frequency behavior of every crystal cut in the singly rotated Y-cut group can be determined from the chart. The chart is a useful tool for the design of frequency standards and precise force-sensing elements. More significantly, the chart provides normalized experimental data to form a basis for a theoretical treatment of the force-frequency effect.

Introduction

The quartz crystals of interest to this study are widely used as resonators in precision frequency standards and for frequency control.¹ The mechanical vibrations set up in the crystal by an alternating electric field (converse piezoelectric effect) react back to the generator of the field (direct piezoelectric effect), thus controlling the frequency of the generator to a high degree of precision.

The resonant frequency and the vibration mode of the crystal are generally dependent upon the physical configuration and the crystallographic orientation (rotation or cut) of the plate, and upon the number, shape, and location of the electrodes. For a particular crystal type, the resonant frequency is further dependent upon the environmental conditions. Among these conditions, the thermal and mechanical disturbances of the plate are of major interest.

To minimize the mechanical disturbances caused by the crystal holder, the frequency sensitivity of the vibrating plate to externally applied forces has been investigated by many users of the crystal. The force-frequency sensitivity was for a long time, however, regarded as an unwanted phenomenon.

The magnitude of the force-frequency effect was established in the last decade and found to be significant and usable. The first application of the effect was the use of external forces applied to the plate for compensation of the

frequency deviations in precision frequency standards operating in an extended temperature range.^{2,3} The force-frequency effect was further investigated at IBM for an application in digital inertial sensors.⁴ In these sensors, an externally applied force to the plate produces frequency deviations which are easily converted electronically into any desired pulse form. During the inertial sensor study, crystal design parameters and force-frequency coefficients were established,⁵ based on normalized experimental results obtained by investigators at IBM and elsewhere.

For frequency control applications, a multitude of crystal cuts and plate configurations has been developed and manufactured. Singly and doubly rotated crystals were investigated and ample design and performance data are available. A singly rotated AT-cut, $(y \times l) + 35^\circ$, is widely used in precision frequency standards, computer clocks and other frequency control applications. In the IBM-developed inertial sensors, AT-cut crystals were primarily used for the operation of the sensor and to study the behavior of the crystal as a force sensing element. Other crystal cuts were only briefly reviewed and experimentally studied.

A graphical chart is presented in this paper to show the normalized force-frequency coefficient, K_f , for all singly rotated crystals in the rotated Y-cut group, $(y \times l) \theta$, and for force directions in azimuth angles ψ . The coefficient K_f

refers to crystals which are in the form of a circular disk and vibrate at frequencies from 3 MHz to 30 MHz, fundamental or overtone. The vibration mode is thickness-shear. The force applied to the crystal is parallel to the major faces of the plate.

Review of previous work

Stress-induced frequency perturbations in vibrating crystals were first noticed by Bottom.⁶ The perturbations were negative for AT-cut and positive for BT-cut crystals.⁷ The dependence of the force-sensitivity on orientation of the plate was thus established. Bottom also observed that the frequency deviations were linearly proportional to the force applied up to strains approaching the crystal breaking point.^{8,9} The relationship between the direction of compressional force and force-sensitivity was investigated by Gerber^{1,2} and by Mings.¹⁰ Positive, negative and zero force-sensitivity directions, identified by the azimuth angle ψ , were established. The linear proportionality applies to tensional as well as to compressional forces.^{5,11} The frequency deviations under tension and compression are of identical magnitude, but of opposite sign. For a force applied in the crystallographic X-direction, the crystal frequency decreases under tension and increases under compression.

The dependence of force sensitivity of singly rotated crystals of the rotated Y-cut group ($y \times l$) on the orientation angle θ and the direction of force ψ has been established by Ballato and Bechmann.^{12,13,14} The force-sensitivity was defined for a particular crystal configuration, subjected to compressional forces, by a coefficient:

$$P_f = \frac{\Delta f}{\Delta F} \frac{1}{f_0} (10^{-8} \text{ g}^{-1}), \quad (1)$$

where f_0 is the fundamental frequency of vibration, ΔF is the change in force. The force-frequency sensitivity is further proportional to the crystal operating frequency, f , either fundamental or overtone.⁵

As a byproduct of the investigations on the suitability of the vibrating quartz crystals for digital sensors, two new parameters, D and η , were introduced as a means of normalizing the data obtained by various investigators of force sensitivity. For one, Farrell¹⁵ found experimentally that the frequency deviations at the same force level are inversely proportional to the diameter, D , of the crystal plate. The other parameter, η , was introduced to account for the effects of crystal holders and differences in experimental equipment. The need for the factor resulted from the observation that frequency deviations under tension and compression for identical applied force were not equal. The explanation for this phenomenon is the assumption that the crystal plate under compression is also subjected to bending^{11,16} and to unwanted distortions caused by the crystal holder.⁵ This is not the case for tensional forces.

With the parameters Δf and η established experimentally, the force-frequency sensitivity, F_s , of a particular crystal specimen is expressed by⁵

$$F_s = \frac{\Delta f}{F} = \frac{K_f f^2 \eta}{nD}, \quad (2)$$

where n is the order of overtone. The coefficient is, therefore

$$K_f = \frac{\Delta f}{F} \frac{nD}{f^2 \eta}. \quad (3)$$

For thickness modes the frequency is inversely proportional to plate thickness, t . The normalized coefficient K_f is therefore proportional to the unnormalized coefficient, P_f , (Eq. (1)) as follows:

$$K_f \propto P_f (Dt), \quad (4)$$

where (Dt) is the geometry factor equal to the cross-sectional area at the center of the plate.

On the basis of experimental data a value of the coefficient for AT-cut crystals ($\theta = +35^\circ$), with force applied in the direction $\psi = 15^\circ$, was found to be:⁵

$$K_f = -23.3 \times 10^{-15} (\text{msN}^{-1}). \quad (5)$$

This value is constant within experimental errors for all AT-cut circular crystals of various plate diameters, subjected to various amounts of compressional force, and operating at frequencies from 3 to 30 MHz.

The coefficient K_f represents the effect of force upon the frequency, i.e., the effect of stress upon the velocity of wave propagation. Because the elastic constants vary with crystallographic orientation, the coefficient K_f may also be expected to vary for crystal cuts other than the AT-cut and for directions of force other than $\psi = 15^\circ$. This has been confirmed by Keyes and Blair.¹⁷

According to Toupin's theory,^{18,19,20} the difference between the squares of velocity, v , of wave propagation through a quartz plate subjected to a uniform stress field, $\sigma_{\psi\psi}$, and velocity, v_0 of propagation through an unstressed quartz plate is given by

$$(v^2 - v_0^2) = A(\theta, \psi) \sigma_{\psi\psi}, \quad (6)$$

where $A(\theta, \psi)$ is a constant dependent on the angle of cut θ and the azimuth angle ψ . When a uniform compressional stress field $\sigma_{\psi\psi}$ is produced by two diametrically opposed forces F , the stress problem determines that $\sigma_{\psi\psi}$, in turn, is proportional to F :

$$\sigma_{\psi\psi} = B(\theta, \psi) F, \quad (7)$$

where $B(\theta, \psi)$ is a constant dependent also on θ and ψ . Combining Eqs. (6) and (7) yields

$$(v^2 - v_0^2) = [A(\theta, \psi) B(\theta, \psi)] F. \quad (8)$$

Using the third-order elastic constants, published by Thurston and Brügger,²¹ a satisfactory correlation was obtained

Table 1 Summary of data sources used for calculation of K_f

References				Crystal parameters*										Assumed or interpolated parameters
Ref. No.	Rep't No.	Page	Fig.	Appl. force	Δf	n	D	F	f	η	θ	ψ	Others	
2		245	1	comp.		×		×	×			×	$\Delta f/f$, AT-cut	$D = 0.550$ in., $\eta = 0.98$
3		56	1	comp.								×	$\Delta f/fF$, AT-cut	$n = 1$, $D = 0.550$ in., $f_0 = 10$ MHz, $\eta = 0.98$
9		45	V. 2	comp.	×			×	×			×	AT-cut	$n = 1$, $D = 0.537$ in., $\eta = 0.9$
	1	22 23	5 6	comp.		×	×	×	×			×	AT-cut	$\eta = 0.9$, Δf
10	2	19 20	3 4	comp.	×	×		×	×			×	AT-cut, D (calc.)	$\eta = 0.9$, D/t
	4	23 25 26	10 11 12	comp.	×	×		×	×			×	BT-cut	$D = 0.500$ in., $\eta = 0.9$
12 13 14				comp.		×	×	×			×	×	f_0, P_f , AT-cut BT-cut Y-cut	$\eta = 0.995$
	12	21	2-4	tens.		×		×	×			×	$\Delta f/f$, D/t , AT-cut, D (calc.)	$\eta = 1$
16	13	35	2-7	tens.		×		×	×			×	$\Delta f/f$, D/t , AT-cut, D (calc.)	$\eta = 1$
	14	20	2-1	tens.	×	×		×	×			×	BT-cut	$D = 0.500$ in., $\eta = 1$
unpub. IBM data				comp. tens.	×	×	×	×	×	×		×	AT-cut	

* The X's indicate parameters for which data is given in the referenced papers.

by Keyes and Blair¹⁷ between the coefficient K_f based on Ballato's¹³ experimental results and calculated values based on isotropic stress theory, i.e., constant B independent of θ and ψ . Correction factors, calculated by Tu,²⁰ take into account the anisotropy of the quartz in the stress problem alone (B is dependent on θ and ψ).

The state of the art of precise measurement of frequency and other electrical crystal parameters has progressed to the point where second-order effects can be demonstrated. Hammond et al²² observed an asymmetry in resonance curves of a quartz crystal driven at high input levels. Apart from effects caused by thermal stress, the behavior of the crystal is attributed to a non-linearity of the elastic constants.

The art of measurement of forces and their control in magnitude and direction has not progressed far. A precise measurement of force still remains an extremely difficult task.^{5,10,11,16} In comparison to electrical measurements, the accuracy of force measurement is lower by at least two orders of magnitude. Hence no conclusive higher order

effects between force and frequency were observed in any experiment reported in this paper.

The behavior of quartz crystal plates of several geometrical configurations, subjected to superimposed forces, was investigated by Mingins.^{1,23} The frequency deviations, caused by the superimposed forces were found to be equal to the algebraic sum of the frequency deviations caused by the component forces.

It appears that no general or precise method for calculating the resonant frequency or the coefficient K_f of quartz crystals in strict accordance with the theory of elasticity is yet known. The force-frequency effect still defies a theoretical explanation.¹

Crystal parameters available for normalization

The experimental results on the force sensitivity of various types of crystals published by Ballato, Bechmann, Gerber, and Mingins, and the results obtained at IBM were compiled and normalized according to Eq. (3). Particularly useful and complete were the data obtained from Ballato.¹⁴

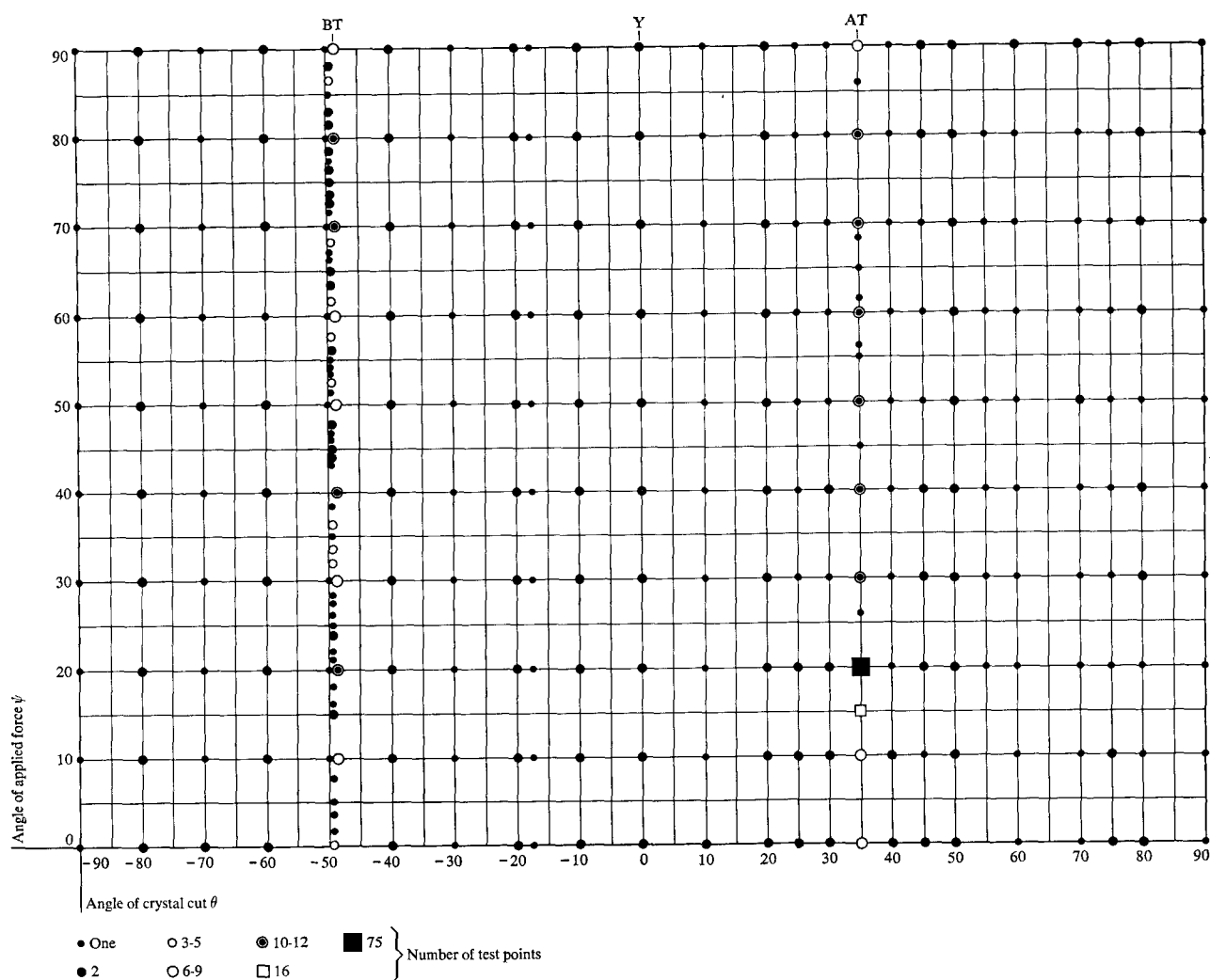


Figure 1 Number of experimental test points available for compilation of K_f graphs.

Most investigators had not considered the importance of the crystal plate diameter in evaluating their results and the exact diameter of the plate was, in many instances, not quoted. When a D/t ratio was given, the diameter was calculated using the thickness of the plate derived from the fundamental resonant frequency. If no value for the diameter was given, the diameter was assumed, taking into account work published elsewhere by the same author. Hence the uncertainty of the exact plate diameter contributed to the wider scattering of the K_f values than the experimental errors would indicate.

The value of the factor, η , used to account for the effect of crystal holders on the force-frequency behavior of the crystal was determined experimentally by the following procedure. A selected AT-cut crystal was mounted in crystal holders of eleven different designs having various arrangements of clips, springs, cemented spots, etc. The crystal was subjected in these holders to compressional forces by a

standard method.⁵ The same crystal type was suspended on cemented wires and subjected to tensional forces. For each holder, the degrading factor η was assumed to be unity when the crystal was subjected to tensional forces. By comparing the results obtained under compression with the results obtained under tensional forces, the allocation of a value for η to a particular holder design was possible. The value of η ranged from 0.8 to 0.995, and the average value for the holders considered was 0.95.

A summary of the crystal parameters and data sources used for the calculation of the coefficient K_f is presented in Table 1. The number of data points available from the literature and from IBM studies for pairs of (θ, ψ) values are shown in Fig. 1.

Graphical presentation of experimental results

The available test results were normalized and values of the coefficient K_f were calculated from Eq. (3). The calculated

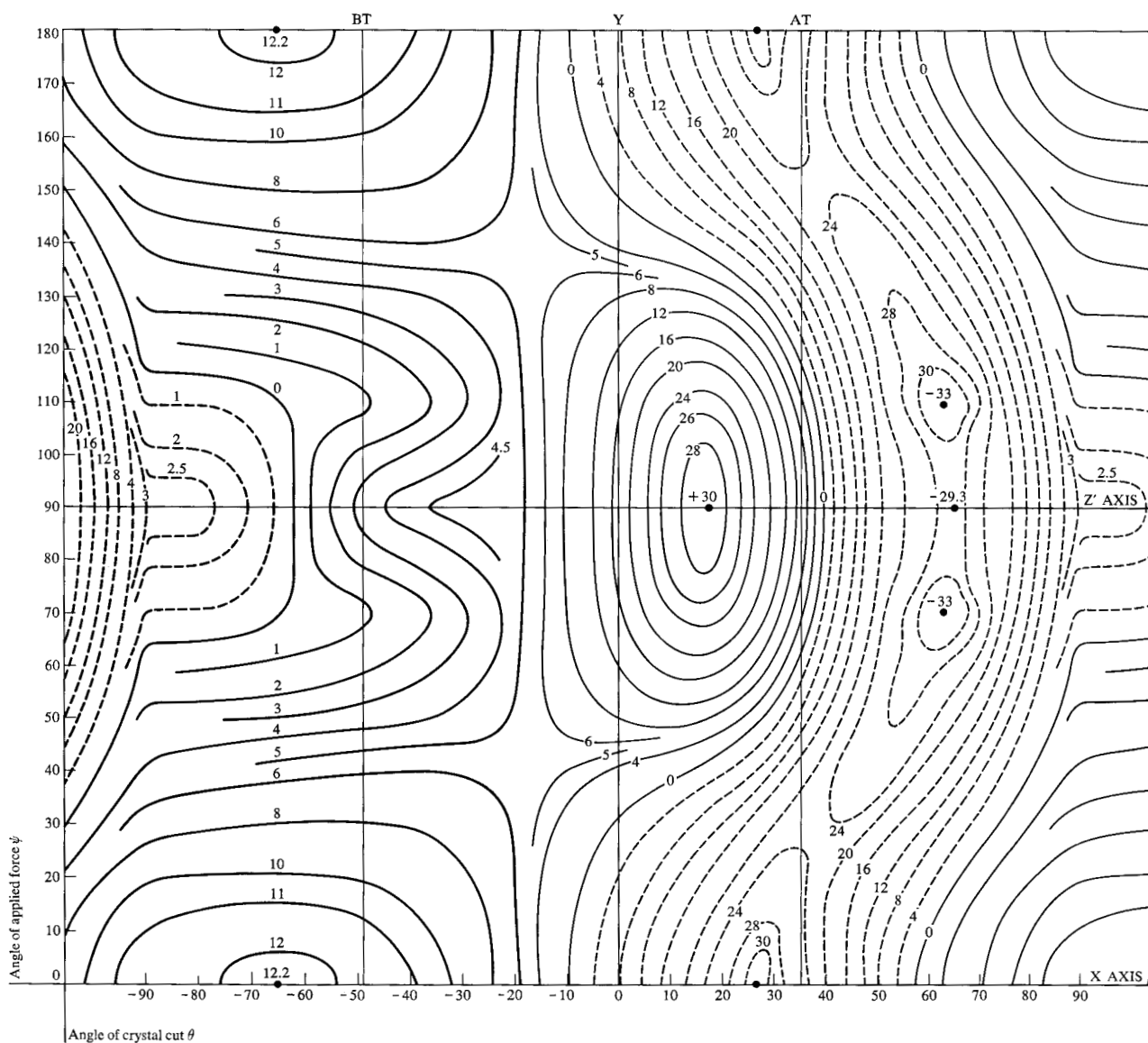


Figure 2 Force-frequency coefficient K_f ($10^{-16} \text{ msN}^{-1}$) as function of orientation angle θ and direction of force, azimuth angle ψ .

values were recorded graphically as points on 55 separate charts that each presented $K_f(\psi)$ for a given value of θ , or $K_f(\theta)$ for a given value of ψ . Whenever there were sufficient data to yield a number of K_f points for a particular pair of (θ, ψ) values, the arithmetic mean of the points was determined. Curves were then drawn on the $K_f(\psi)$ and $K_f(\theta)$ charts through the average K_f values.

The curves as originally drawn were improved by taking the average K_f values at selected pairs of (θ, ψ) coordinates where the value of K_f on the $K_f(\psi)$ chart was different from its value at the same coordinates on the $K_f(\theta)$ chart. The number of such corrections was limited to few points, and the differences were, surprisingly, not large.

The corrected curves were then used as the basis for a single chart in which K_f values are shown as isograms on a plot of θ vs ψ , Fig. 2. On this chart, solid lines refer to positive values of K_f , and dashed lines to negative values, where positive values result from tension forces and the negative values from compression forces. The chart is constructed for the ranges $-90^\circ \leq \theta \leq 90^\circ$ and $0^\circ \leq \psi \leq 180^\circ$, with isograms extended beyond the range of θ for purposes of clarity.

The chart is symmetrical along the line $\psi = 90^\circ$, and shows two distinct patterns, divided by a constant coefficient K_f line between $\theta = -15^\circ$ and $\theta = -20^\circ$ for all azimuth angles ψ . A maximum positive force sensitivity is

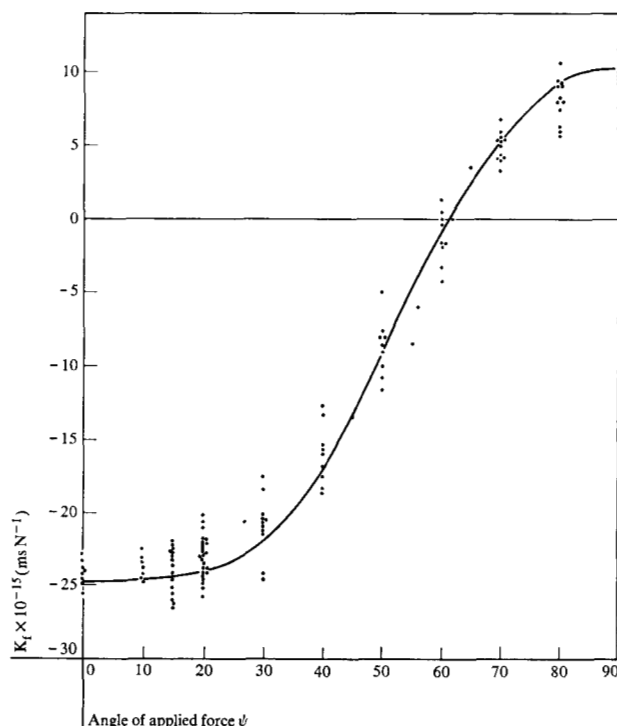


Figure 3 $K_f(\psi)$ for the crystal cut $(y \times l) + 35^\circ$. The curve was derived from Fig. 2, and the dots represent normalized data points obtained from studies on AT-cut crystals. $K_f(\psi = 0^\circ) = -24.7 \times 10^{-15} \text{ (msN}^{-1}\text{)}$, $K_f(\psi = 90^\circ) = +10.3 \times 10^{-15} \text{ (msN}^{-1}\text{)}$, and $K_f = 0$ at $\psi = 61.3^\circ$.

shown by a crystal cut $(y \times l) + 17^\circ$ with a coefficient:

$$K_f \cong +30 \times 10^{-15} \text{ (msN}^{-1}\text{)} \text{ at } \psi = 90^\circ. \quad (9)$$

Three negative maxima appear with the following coefficients:

$$K_f \cong -30.8 \times 10^{-15} \text{ (msN}^{-1}\text{)} \text{ at } \theta = +26.5^\circ, \psi = 0^\circ, \quad (10)$$

$$K_f \cong -33 \times 10^{-15} \text{ (msN}^{-1}\text{)} \text{ at } \theta = +63^\circ, \psi = 70^\circ, \quad (11)$$

$$K_f \cong -29.3 \times 10^{-15} \text{ (msN}^{-1}\text{)} \text{ at } \theta = +65^\circ, \psi = 90^\circ. \quad (12)$$

In certain areas the accuracy of the chart is doubtful, particularly where the K_f graphs are based on one experimental point or are interpolated. For example, no experimental data are available for a crystal cut $(y \times l) + 65^\circ$ (see Fig. 1). The positions of the two negative maxima (Eqs. (11) and (12)) are based on interpolated data only. It is quite possible that one negative maximum is located at $\psi = 90^\circ$. To confirm this, additional experiments are needed with crystals cut to precise angles around $\theta = +64^\circ$. It is of interest to note that all maxima of force sensitivity lie in the area of positive θ angles.

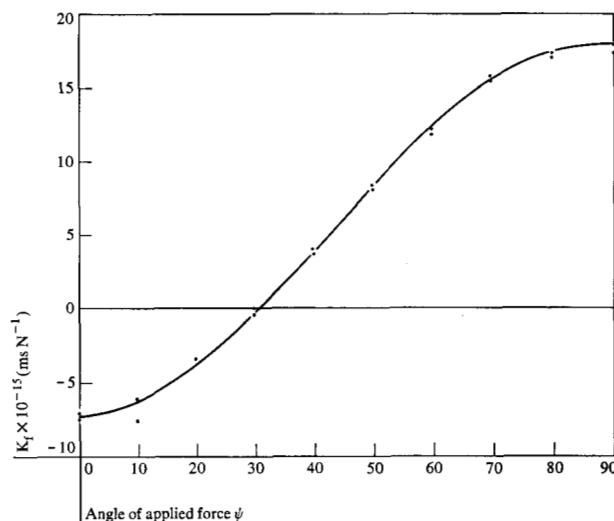


Figure 4 $K_f(\psi)$ for the crystal cut $(y \times l) 0^\circ$. The curve was derived from Fig. 2, and the dots represent normalized data points obtained from studies on Y-cut crystals. $K_f(\psi = 0^\circ) = -7.3 \times 10^{-15} \text{ (msN}^{-1}\text{)}$, $K_f(\psi = 90^\circ) = +17.6 \times 10^{-15} \text{ (msN}^{-1}\text{)}$, and $K_f = 0$ at $\psi = 31^\circ$.

Crystal cuts of interest

A crystal of the orientation $(y \times l) - 18.5^\circ$ shows a uniform force sensitivity with angle ψ and a constant coefficient:

$$K_f \cong 5.3 \times 10^{-15} \text{ (msN}^{-1}\text{)}. \quad (13)$$

The area between $\theta = -40^\circ$ and $\theta = -15^\circ$ requires further experimental investigation, not only to obtain the magnitude of the force sensitivity in this area, but also to ascertain the exact angle θ at which the force sensitivity is constant for all azimuth angles ψ .

Other cuts of interest are the well known AT-, Y-, and BT-cuts. Plots of K_f vs ψ for these cuts, obtained from isograms of the chart, Fig. 2, are shown in Figs. 3, 4, and 5. On the graph for AT-cut crystals, Fig. 3, the solid line refers to the cut $(y \times l) + 35^\circ 0'$. The test results (dots) refer to data from AT-cut crystals without reference to the exact angle of θ . The angle θ of AT-cut crystals can vary between $+35^\circ 8'$ and $+35^\circ 30'$, depending primarily on thermal requirements and order of overtone. The interpolated differences,

$$\Delta K_f(\psi) = K_{f(\theta = +35^\circ 0')} - K_{f(\theta = +35^\circ 30')}, \quad (14)$$

are shown in Fig. 6. For certain angles of θ , a small variation in the angle can introduce large variations of the coefficient K_f and in the force-frequency behavior of the crystal. As illustrated by the isograms of the chart, Fig. 2, the angle θ , as well as the angle ψ , is a commanding factor affecting the force-frequency coefficient. Small variations in the angle θ for the crystal cuts investigated have not been experimentally explored. Hence, the unknown deviations

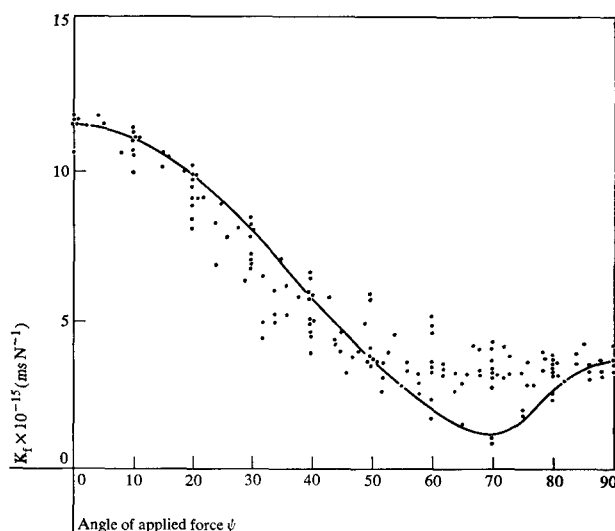


Figure 5 $K_f(\psi)$ for the crystal cut ($y \times l$) -49° . The curve was derived from Fig. 2, and the dots represent normalized data points obtained from studies on BT-cut crystals. A number of test points, particularly for ψ -angles between 50° and 80° , were not used for curve-fitting because they were judged to contain large errors caused by the experimental set-up at the low force-frequency sensitivities recorded. $K_f(\psi = 0^\circ) = +11.5 \times 10^{-15}$ (msN^{-1}), $K_f(\psi = 90^\circ) = +3.4 \times 10^{-15}$ (msN^{-1}), and $K_{f \min}$ (at $\psi = 70^\circ$) $\cong 1 \times 10^{-15}$ (msN^{-1}).

in the angle θ of the specimen tested are one of the factors contributing to the scattering of the test results. The direction of force, azimuth angle ψ , was set and measured within $\pm 1^\circ$ in all experiments considered.

Conclusions

A chart based on normalized experimental results has been derived to illustrate the dependence of force sensitivity and its coefficient K_f upon crystallographic orientation of the crystal plate and upon the direction of diametrically opposed force applied to the plate. The chart illustrates the relation between the coefficient K_f and the combined effects of elastic constants of crystalline quartz; hence, it can provide a basis for a theoretical treatment of the force-frequency effect.

The force sensitivity of any crystal in the rotated Y-cut group can be obtained from the chart for evaluation of crystal holder effects or for a design of force sensing elements for various transducers based on the phenomenon.

The chart should be used with caution as it is derived entirely empirically and is based on a limited number of experimental data and on interpolations. Furthermore the exact values for the crystal parameters used for normalization of test results were, in many instances, not known. The chart is subject to corrections as further experimental and, perhaps, theoretical data become available.

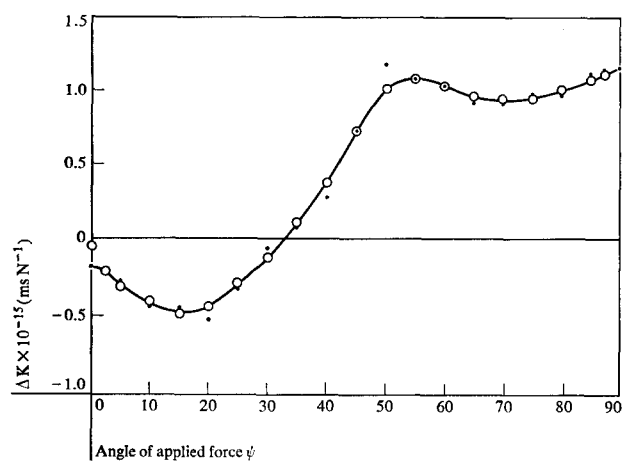


Figure 6 Interpolated differences ΔK_f as function of angle ψ , where $\Delta K_f = K_f(\theta = 35^\circ \text{ or } 0^\circ) - K_f(\theta = 35^\circ \text{ or } 90^\circ)$. Dots represent differences taken from K_f graphs and isograms of Fig. 2. Circles represent arithmetic means of fitted curve.

A maximum force-frequency coefficient of all crystals in the rotated Y-cut group is approximately:

$$|K_f| \cong 30 \times 10^{-15} \text{ (msN}^{-1}\text{)}$$

for angles $\theta = +17^\circ$, $+26.5^\circ$ and $+64^\circ$ located at azimuth angles $\psi = 0^\circ$ and at, or near, $\psi = 90^\circ$. Of interest is the crystal cut with θ near -18° which shows a constant coefficient:

$$K_f \cong 5.3 \times 10^{-15} \text{ (msN}^{-1}\text{)}$$

over the whole range of angles ψ . The coefficient $K_f(\psi)$ is also shown graphically for the AT-, Y-, and BT-cut crystals with values given for the maxima of K_f and the location of zero force sensitivity.

It is hoped that the chart and the graphs provide at least a pictorial presentation of the force-frequency phenomenon and may prove to be of value for a theoretical treatment in strict accordance with the theory of elasticity.

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